Cooling degree-days and their applicability to building energy estimation

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List of Symbols

A	Area (m^2)
C	Thermal capacity of fabric, $c_{p,f}\rho V_f$ (kJ K^{-1})
c_p	Specific heat capacity of air (kJ kg^{-1} K^{-1})
$c_{p,f}$	Specific heat of building fabric (kJ kg^{-1} K^{-1})
COP	Coefficient of performance
D_c	Cooling degree-days (K-day)
E	Building energy demand (kWh)
$\hat{E}$	Degree-day estimated energy demand (kWh)
FAF	Fresh air fraction
g_s	Moisture content of supply air (kg /$kg_{dry\ air}$)
g_o	Moisture content of outside air (kg /$kg_{dry\ air}$)
h_{fg}	Enthalpy of vaporisation of water (kJ kg^{-1})
k	Constant
$\dot{m}$	Mass flow rate of air (kg/s)
N	Number of air changes per hour (h^{-1})
n	number of days in the month
ΔP	Pressure rise across fan (kPa)
Q_C	Heat flow into thermal storage (kW)
Q_{fabric}	Heat gain through the building fabric (kW)
$Q_{f.a,S}$	Sensible fresh air load (kW)
Q_{fan}	Heat gain across the supply air fan (kW)
$Q_{f.a,L}$	Latent fresh air load(kW)
Q_I	Internal sensible heat gains to the building (kW)
Q_L	Latent heat gains into the building (kW)
Q_L'	Total effective latent gain (kW)
Q_S	Sensible heat gains to building (Q_{solar} + Q_I)(kW)
Q_{solar}	Solar heat gains into the building (kW)
t	Time (h)
U	Building fabric U value (W m^{-2} K^{-1})
U'	Building overall heat loss coefficient, $(\Sigma UA+{}^{1}/_{3}NV)/1000$ (kW K^{-1})
V	Volume of space (m^3)
V_f	Volume of fabric (m^3)
$\dot{v}$	Volume flow rate of air (m^3 s^{-1})

Greek symbols

ε	Heat exchanger effectiveness
$\Delta\theta_i$	Change in internal temperature (K)
$\Delta\theta_L'$	Fictitious temperature rise from latent gains (K)
η_{fan}	Fan efficiency
λ	Rate of heat transfer parameter (kW K^{-1})
θ_{ai}	Internal temperature (°C)
θ_{ao}	Outside temperature (°C)
θ_{ao}'	Mean outside temperature during occupied hours (°C)
θ_b	Base temperature (°C)
θ_c	Off coil temperature (°C)
θ_{eo}	Sol-air temperature (°C)
θ_m	Mixed or recovered air temperature (°C)
θ_r	Return air temperature (°C)
θ_s	Supply air temperature (°C)
θ_{sp}	Control set point temperature (°C)
ρ	Density of building fabric (kg m^{-3})
σ_{go}	Standard deviation of outside moisture content (kg/ $kg_{dry\ air}$)
τ	Building time constant, C/U' (h)

Superscripts

$\bar{\ }$	Mean or averaged value

Introduction

The energy performance of buildings has taken on a greater significance with the setting of domestic and international targets for the reduction of greenhouse gas emissions. Energy use is the largest contributor to these emissions. Simple building energy estimation techniques have a role in promoting energy conscious design and operation, but they are imperfect tools. Recent work has presented a more rational development of heating degree-days, including the uncertainty associated with estimates of energy demand [1][2][3]. However, no satisfactory treatment of cooling degree-days has been adopted for air-conditioned or cooled spaces. Note that in this paper only the energy consumed by mechanical refrigeration plant is considered, and not the other energy consuming items such as the fans, re-heaters or humidifiers.

This paper applies principles used in the development of heating degree-day models to the cooling degree-day problem, with particular emphasis placed on the validity of a base temperature. For heating applications the base temperature is defined as the outside temperature at which the heating system does not operate, and is derived from dividing heat gains by the building heat loss coefficient. Similarly the base temperature for cooling applications can be defined as the outside temperature at which the refrigeration plant need not operate to meet the space cooling requirements. If dealing only with sensible gains this would be equal to the off-coil supply air temperature. This can be calculated by dividing the gains to the space by the sensible heat capacitance ($\dot{m}c_p$) of the air, and subtracting the resulting temperature difference from the room set point temperature. The gains to the space are dominated by direct gains from solar radiation, people and machines and lights. Whilst gains through the building fabric will be a contribution they will form small fraction of the load, which renders the approach adopted for heating degree-days inappropriate. This new approach to defining a cooling degree-day base temperature is adopted in this paper, with necessary modifications to account for latent heat gains and typical heat recovery arrangements.

The literature on cooling degree-days is lacking a rigorous theoretical treatment. In most cases cooling degree-days continue to employ traditional definitions of a heating base temperature [4]. Such claims have been supported by empirical evidence using regression analysis; but the results cannot be held to have general applicability in the absence of a theoretical foundation. This paper addresses this gap by examining the heat balances for typical air-conditioning systems to show how the base temperature can be more appropriately defined.

Theory

The principle of cooling degree-days is similar to heating degree-days, in that the degree-day integral is proportional to the building energy balance over a stated period of time. The energy demand on the cooling system will therefore be the cooling degree-days multiplied by some heat transfer parameter, which for the moment will be given the term λ

$$\hat{E} = \lambda \int (\theta_{ao} - \theta_b) dt \qquad (1)$$

For $\theta_{ao} > \theta_b$.

Previous descriptions of cooling degree-days [4] have proceeded on the traditional basis of taking λ to be the building heat loss coefficient U'. θ_{ao} is the outside temperature and θ_b is the base temperature defined by

$$\theta_b = \theta_{ai} - \frac{Q_S}{U'} \qquad (2)$$

where θ_{ai} is the inside air temperature and Q_S is the combined sensible gain. However, heat transfer through the fabric is normally a minor component of heat gain into a building, and there is little physical justification for such an approach. In this paper cooling degree-days will be defined with respect to the cooling system, in particular the air flow rates and the energy extracted by the cooling coil; the various heat gain components will be separately identified and incorporated into a rational degree-day model.

Figure 1 shows a simple schematic of an air-conditioning system with just a cooling coil and fan; not considered here are re-heating, heat recovery or humidity control.

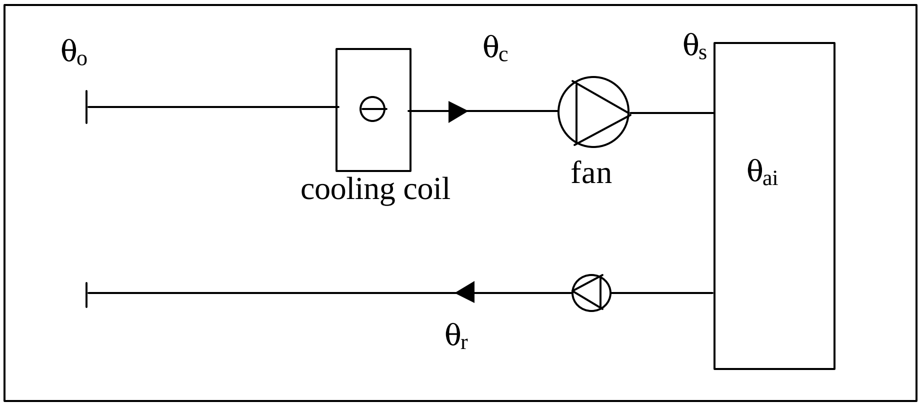

Figure 1 Air-conditioning system with no re-heat / heat recovery/ humidity control

The rate at which energy is extracted from the air by the coil is

$$Q_E = \dot{m}c_p(\theta_{ao} - \theta_c) + \dot{m}h_{fg}(g_o - g_c) \qquad (3)$$

where Q_E is the rate of heat removal from the air, m is the mass flow rate of the air, c_p is the specific heat of air, h_{fg} is the enthalpy of vaporisation of water, θ_{ao} is the outside air temperature, θ_c is the off coil air temperature, and g_o and g_s are similarly the outside and off coil moisture contents respectively. An integration of equation (3) over some period of time would give a definition of cooling degree-days, but would also include the latent load in terms of “moisture-days”. This would require a great deal more meteorological information, including wet bulb temperatures. In order to arrive at a simpler concept it is useful to look at the load in terms of its components.

$$Q_E = Q_{fabric} + Q_{solar} + Q_I + Q_{fan} + Q_{f.a.,S} + Q_{f.a.,L} + Q_L \qquad (4)$$

Where Q_{solar}, Q_I, and Q_L are the solar, internal sensible, and room latent loads respectively. Q_{fabric} is the heat gain through the fabric given by

$$Q_{fabric} = U'(\theta_{eo} - \theta_{ai}) \qquad (5)$$

Where θ_{eo} is the sol-air temperature. $Q_{f.a.,S}$ is the net sensible heat extracted from the fresh air air, given by

$$Q_{f.a.,S} = \dot{m}c_p(\theta_{ao} - \theta_{ai}) \qquad (6)$$

Where $\dot{m}$ is the mass flow rate of air and c_p is the specific heat of air. $Q_{f.a.,L}$ is the latent heat extracted from the air given by

$$Q_{f.a.,L} + Q_L = \dot{m}h_{fg}(g_o - g_s) \qquad (7)$$

Q_{fan} is the heat imparted to the air by the fan

$$Q_{fan} = \frac{\dot{v}\Delta P}{\eta_{fan}} = \dot{m}c_p(\theta_s - \theta_c) \qquad (8)$$

where $\dot{v}$ is the volume flow rate of air, ΔP is the pressure rise across the fan, η_{fan} is the fan efficiency and θ_s is the supply air temperature.

The sensible gains make up the majority of the load. According to Jones [5], the latent gains contribute less than 10% of the total for the theoretical office block. It is therefore reasonable to relate the cooling degree-day base temperature to sensible temperature rises as experienced by the air system. The energy balance can now be expressed as

$$Q_E = \dot{m}c_p\left[(\theta_{ao} - \theta_{ai}) + \frac{\dot{v}\Delta P}{\dot{m}c_p\eta_{fan}} + \frac{Q_S}{\dot{m}c_p}\right] + U'(\theta_{eo} - \theta_{ai}) + \dot{m}h_{fg}(g_o - g_s) \tag{9}$$

The challenge is to incorporate the fabric and latent load terms within the square brackets in a meaningful and reliable way to give an overall temperature difference due to gains. The treatment of the fabric term is incomplete here considering that there may be some positive load through the fabric during the day, but that night-time cooling may have a gain mitigation effect due to thermal capacity; a modification to this will be developed later in this paper. For now a mean daily fabric gain is assumed, in which case the fabric term can be brought within the square brackets by dividing by $\dot{m}c_p$

The latent term is problematic since it does not contain units of temperature difference. The most expedient way to deal with this is to treat the latent as if it were sensible and assign to it a fictitious temperature rise; i.e. the rise that would be experienced by the air flow if it picked up a sensible gain of the same magnitude. The sensible gains are balanced by the room air/supply temperature difference, and the mass flow of air as shown in equation (10)

$$Q_{solar} + Q_I + Q_{fabric} = \dot{m}c_p(\theta_{ai} - \theta_s) \tag{10}$$

A sensible gain equal to the magnitude of the latent gain, Q_L', is added to equation (10) and re-arranged. This gives:-

$$\frac{Q_{solar} + Q_I + Q_{fabric} + Q_L'}{\dot{m}c_p} = \left(\theta_{ai} - \theta_s + \Delta\theta_L'\right) \tag{11}$$

Where $\Delta\theta_L'$ is the fictitious temperature rise due to latent gains. From the definition supplied by equation (10) the terms can be separated to show

$$\frac{Q_L'}{\dot{m}c_p} = \Delta\theta_L' = \frac{h_{fg}}{c_p}(g_o - g_s) \tag{12}$$

Taking typical values of h_{fg} and c_p at 2450 kJ kg^{-1} and 1.02 kJ $kg^{-1}K^{-1}$ respectively gives

$$\Delta\theta_L' = 2400(g_o - g_s) \tag{13}$$

which demonstrates that this temperature rise is independent of the mass flow rate of air. It is now possible to combine all the components of equation (9) to give

$$Q_E = \dot{m}c_p\left[\left(\theta_{ao} - \theta_{ai}\right) + \frac{\dot{v}\Delta P}{\dot{m}c_p\eta_{fan}} + \frac{Q_S}{\dot{m}c_p} + \frac{U'}{\dot{m}c_p}\left(\theta_{eo} - \theta_{ai}\right) + 2400\left(g_o - g_s\right)\right] \qquad (14)$$

If assuming an average fabric gain, and expressing in terms of cooling degree-days, gives

$$\int Q_E\, dt = \dot{m}c_p \int \left\{\theta_{ao} - \left[\theta_{ai} - \frac{\dot{v}\Delta P}{\dot{m}c_p\eta_{fan}} - \frac{Q_S}{\dot{m}c_p} - \frac{U'}{\dot{m}c_p}\left(\theta_{eo} - \theta_{ai}\right) - 2400\left(g_o - g_s\right)\right]\right\} dt \qquad (15)$$

The term within the square brackets equates to the base temperature. There remain the issues of evaluating the latent component, incorporating a thermal capacitance effect and a treatment of heat recovery.

Determination of latent component

Moisture content of atmospheric air varies widely throughout the year and shows significant diurnal swings. There is no meaningful relationship between moisture content and dry bulb temperature, nor is there a relationship between sensible and latent gains. However, it can be argued that due to the characteristics of the cooling coil, the change in air moisture content across the coil will show a small variability throughout a month; it is therefore reasonable to assume the use of a monthly average latent load on the coil. (The use of average monthly temperatures for intermittent effects has been shown to be reasonable practice for heating applications [3], and it will be assumed here that this concept is transferable to moisture content).

This requires an estimate of the mean monthly moisture difference across the cooling coil for those occasions when $g_o > g_s$. If the off-coil moisture content is assumed to be reasonably constant (which may be a reasonable assumption where humidity control is not important) then this mean moisture difference is dependent on the mean moisture content $\bar{g}_o$. However, on a number of occasions within a month $g_o < g_s$, and the distribution of $(g_o - g_s)$ is a truncated form of the distribution of g_o (for an explanation of truncated distributions see for example Thom [6]). Hitchin [7] has shown that for temperature, such truncated distributions are well described by an expression using only mean monthly temperature and the standard deviation of the temperature. It has been found here that this formula is also applicable to moisture content, and the mean positive moisture difference across the coil can be found by

$$\overline{\left(g_0 - g_s\right)} = \frac{\bar{g}_o - g_s}{1 - e^{-k\left(\bar{g}_o - g_s\right)}} \qquad (16)$$

where $\bar{g}_o$ is the mean monthly outside moisture content and

$$k = \frac{2.5}{\sigma_{g_o}} \qquad (17)$$

An analysis of ten years of weather data for London conducted for this paper revealed that equation (16) shows a good fit to real data when k is defined by equation (17). This analysis showed that the mean value of k for summer months is 1700, with maximum and minimum values of 2300 and 1220 respectively. Figure 2 shows the sensitivity of the fictitious temperature rise, $\Delta\theta_L'$, to variation in k for $\bar{g}_o$ = 0.00925 kg kg^{-1} and $\bar{g}_s$ = 0.008 kg kg^{-1}. It can be seen that for the range of k given above the fictitious temperature rise varies by less than 0.7°C.

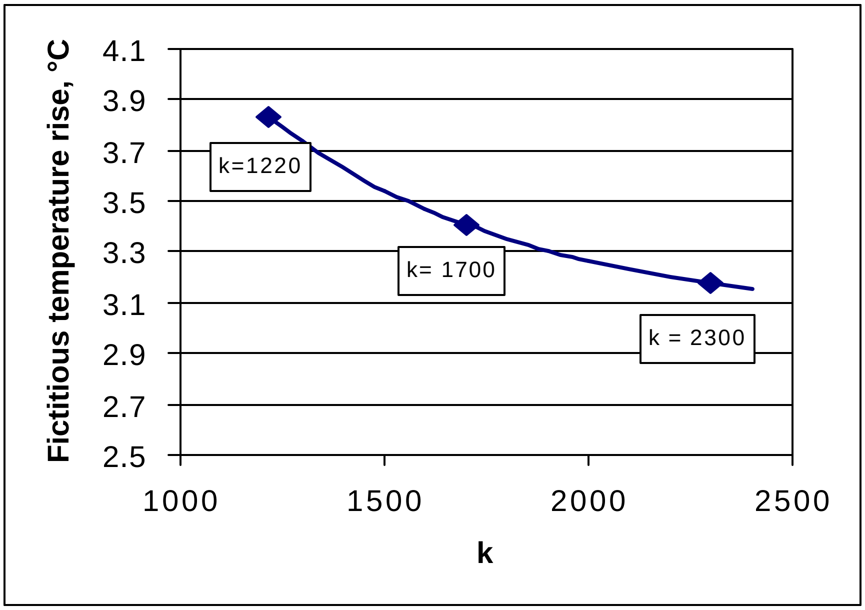

Figure 2 Sensitivity of fictitious temperature rise to variation in k

Heat recovery

Figure 3 shows the same simple system as Figure 1, but includes a heat recovery device. Sensible heat recovery only will be considered in this paper, in particular the case where return air is cooler than in-coming outside air; this will result in a lower sensible load on the cooling coil. In this case θ_{ao} in equation (15) is replaced by the on-coil temperature θ_m, which can be expressed in terms of θ_{ao}, θ_r (the return air temperature) and the effectiveness of the heat exchanger.

The effectiveness, ε, of a counter flow heat exchanger is constant and is given by

$$\varepsilon = \frac{(\theta_{ao} - \theta_m)}{(\theta_{ao} - \theta_r)} \tag{18}$$

Where θ_r is the return air temperature (normally the room air temperature plus a fan gain rise) and the on-coil air temperature, θ_m, is therefore

$$\theta_m = \theta_{ao} - \varepsilon(\theta_{ao} - \theta_r) \tag{19}$$

Note that for an air mixing heat recovery system ε is equal to (1-FAF), where FAF is the fresh air fraction. However, with an air mixing system a further problem arises in that there will be latent heat recovery, and g_o will be similarly affected; to incorporate this would introduce a new level of complexity and will be considered in later work once the treatment of sensible heat recovery has been verified.

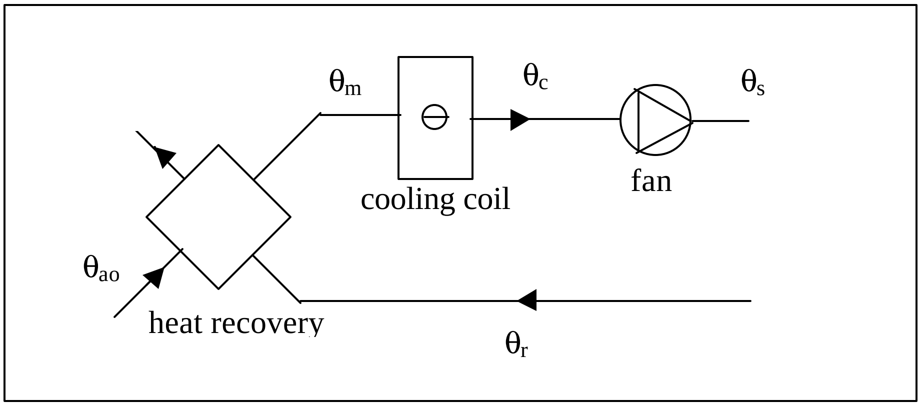

Figure 3 Air-conditioning system with heat recovery

Fabric gains and thermal capacity effects

Although a relatively small component, the sensible gains through the fabric are perhaps the most complex part of the problem due to the timing of the gains, and the effect of thermal storage. Gains through opaque fabric components will be due in part to solar warming of the outside surface, and therefore be related to sol-air temperature, while conducted gains through glazed components will be related to outside air temperature. Also the use of a mean outside air (or sol-air) temperature may give inaccurate results, as a net loss may be shown, without the effects of thermal capacity taken into account. In order to account for positive gains during the daytime, potential night time cooling, and thermal storage of the fabric it is proposed here to follow a similar procedure to that set out for heating degree-days [3].

Firstly to deal with daytime gains it is proposed to use equation (5), but replacing θ_{eo} with θ_{ao}', a mean air temperature related to occupied hours only. The use of air temperature is justified by the fact that in summer due to a small temperature difference, the fabric gains are small in comparison to other gains. Much of the fabric gain is attributed to glazing, and some of the building will be in shade anyway. To capture at least a portion of this gain in a simple procedure is more important than developing a detailed complex model.

The effect of thermal storage is likely to be one of gain mitigation. The thermal store (fabric mass) will lose heat overnight when the gains are zero; the store will thus be available to absorb this amount of heat the following day. This will be experienced as a reduction in load on the cooling coil. The principle of Newtonian cooling can be used to show

$$\Delta\theta_i = e^{-\frac{\Delta t}{\tau}}\left(\theta_{sp} - \theta_o\right) \qquad (20)$$

Where $\Delta\theta_i$ is the change in the temperature of the building fabric, Δt is the unoccupied period (or plant off period) in hours, θ_{sp} is the room set point temperature and τ is the building time constant given by

$$\tau = \frac{\rho c_{p,f} V_f}{U \times 3600} \qquad (21)$$

ρ is the fabric density, $c_{p,f}$ is the specific heat of the fabric and V_f is the effective volume of the building fabric. This change in fabric temperature can be multiplied by the thermal capacity of the structure, and divided by 24 × 3600 to give the average rate of gain that will be absorbed by the structure, Q_C, thus

$$Q_C = \frac{\rho c_{p,f} V_f \Delta\theta_i}{24 \times 3600} \tag{22}$$

The sign of Q_C will be the opposite of the gains, and can be incorporated into the expression for cooling degree-days.

The full expression

Modifying equation (15) in accordance with the foregoing discussion gives

$$D_c = \int \left(\theta_{ao} - \varepsilon(\theta_{ao} - \theta_{ai}) - \theta_b\right) dt \tag{23}$$

With

$$\theta_b = \theta_{ai} - \frac{\dot{v}\Delta P}{\dot{m}c_p \eta_{fan}} - \frac{Q_S}{\dot{m}c_p} - \frac{U'}{\dot{m}c_p}\left(\theta_{ao}{}' - \theta_{ai}\right) - 2400\overline{(g_o - g_s)} + \frac{Q_C}{\dot{m}c_p} \tag{24}$$

The electricity consumption at the chiller can now be found from

$$F = \frac{\dot{m}c_p D_c}{COP} \tag{25}$$

Where COP is the coefficient of performance of the chiller. In practice the COP is dependent on either outside dry or wet bulb temperature, depending on the type of heat rejection equipment used, and the importance of this may vary from system to system. A methodical treatment of this is beyond the scope of this paper.

The base temperature is now seen to be dependent on a number of variables; an initial sensitivity analysis reveals that the most important of these are sensible and latent gains, and inside and supply air temperatures. Fan gain is important, but this is likely to be constant for most systems. Figure 4 shows how base temperature varies with gains. For this illustration typical values of heat loss coefficient, thermal capacity and design operating conditions are assumed. The high gain scenario assumes 100% design sensible and latent gains, where latent gains are as high as 40% of the total load; the low gain scenario assumes 25% of the design gains, and the medium gains case is halfway between the other two. This indicates the importance of calculating the correct base temperature, since this will dictate the degree-day total, and hence the energy estimate. Variation of base temperature due to the influence of the other parameters will require systematic study, but this should be in the context of real (or simulated) building operation in order to be meaningful. Such a study will be part of future investigations in this work.

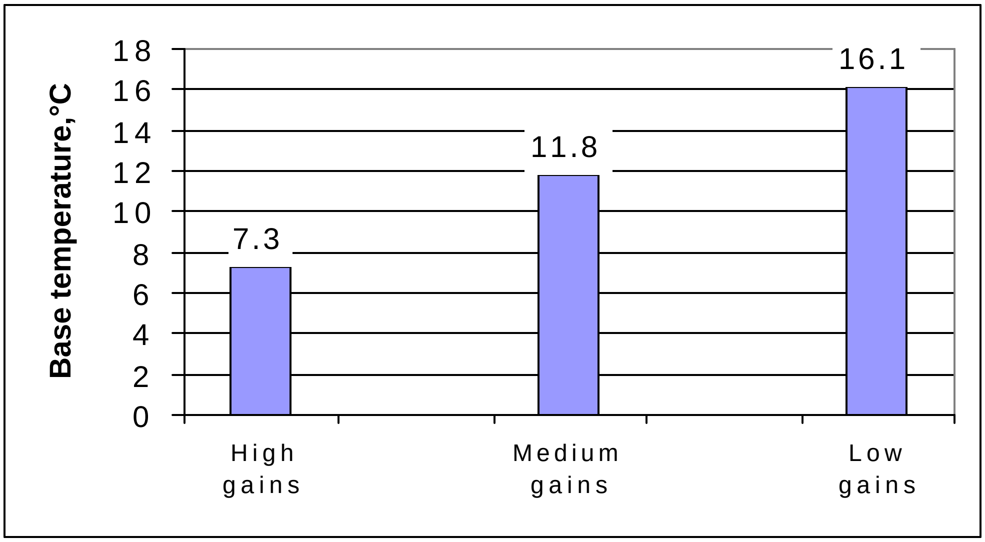

Figure 4 Change in base temperature against variation in sensible and latent gains

Experience with heating degree-days shows that fixing θ_b using average monthly gains and internal temperatures, and conducting the integration using hourly outside temperatures over a month, is appropriate. Taking average monthly values of θ_b for this length of time avoids undue influences of intermittency effects or of peak gains. A month is also a period of time that has identifiable seasonal characteristics, and the plant is likely to operate under similar conditions throughout the period.

Discussion

Equations (23) to (25) form a rational definition of cooling degree-days based on the energy balance of a cooled building. No allowance is made for over-cooling, necessary re-heat, or overriding humidity control. It may be argued that equation (24) gives an overly complex definition of base temperature, given that the aim is to develop a simple model. However, each of the components has an impact on building energy performance, and there is no other rational way in which to combine any of these as they arise from distinctly different influences and processes.

The applications of degree-days are generally considered two-fold; firstly to estimate future energy demand, and secondly to monitor energy performance. For cooling applications frequency of occurrence (or Bin) methods have been adopted with some success. Such frequency of occurrence techniques are more reliable for cooling rather than heating applications due to the coincidence of outside conditions with plant running times, and the reduced importance of thermal capacity effects (a discussion of the importance of thermal capacity in heating applications can be found in [1]). Cooling degree-days may therefore be better suited to monitoring applications. Heating degree-days have been used extensively for such practices and it is suggested that a more rational definition of cooling degree-days may increase their profile with energy managers.

A modern air-conditioned building is likely to be equipped with a building energy management system (BEMS) that can monitor and collect data for all the relevant variables: θ_{ao}, θ_{ai}, g_o, g_s, and solar radiation. For variable air volume systems it would be further necessary to measure the mass flow rate of air, and perhaps apply a mean flow rate to the relevant equations. This is true also of VRV and Versatemp cooling systems. In addition BEMS systems have facilities to automatically analyse the data (normally via a spreadsheet). The equations in this paper can be written in as simple algorithms to calculate cooling degree-days, and hence the theoretical energy demand that the building should use. Cooling degree-days as defined here could be used to develop individual benchmarks for air-conditioned buildings. In theory the degree-day based estimate could be calculated for the ideal set of operating conditions (optimum supply air temperature and flow rate), which could define the minimum energy consumption a building would have under different load regimes. Actual energy consumption can then be compared against these benchmark targets.

Conclusions

This paper has set out a theoretical basis for an improved definition of base temperature for cooling degree-days. This concept extends work that has been carried out with heating degree-days. By applying simple energy balance equations across the building and major plant components a simple degree-day model has been achieved. The model has been adapted to consider the effect of latent heat variations, systems with heat recovery and free cooling, as well as the implications of fabric thermal storage. Some approximations have been made that have allowed certain variables to be approximated as monthly average values and these assumptions have been justified.

Cooling degree-days offer significant advantages by way of simplicity to designers and operators of building services systems. The development of a rational cooling degree-day model could enable better benchmarks to be developed for air-conditioned buildings, allow improved BEMS operation and produce better energy estimating methods. These three factors could make a real contribution to energy efficiency and CO_2 emission targets.

It is proposed to follow up this work in two ways: firstly by using data from simulations of air-conditioned buildings to validate the energy balance; secondly by applying the theory to real building data. Further work would also be required to establish degree-day models for alternative configurations including variable flow systems such as VAV, VRV and Versatemp.

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