Appendix 5.A8: Derivation of thermal steady state model

5.A8.1 Notation

Symbols used in this appendix are as follows.

A_n Area of surface n (m^2)
a, b Linearising constants
b_n Radiant heat transfer coefficient ($W \cdot m^{-2} \cdot K^{-1}$)
c_p Specific heat capacity of air ($J \cdot kg^{-1} \cdot K^{-1}$)
c_v Ventilation conductance ($W \cdot K^{-1}$)
E_{bn} Black body radiation from surface n ($W \cdot m^{-2}$)
F_a fraction of air temperature detected by sensor (0.5 for a sensor detecting operative temperature)
$F_{m,n}$ View factor from surface m to surface n
h_a Heat transfer coefficient between air and environmental nodes ($W \cdot m^{-2} \cdot K^{-1}$)
H_c Thermal transmittance due to convection ($W \cdot K^{-1}$)
h_c Convective heat transfer coefficient ($W \cdot m^{-2} \cdot K^{-1}$)
h_{cn} Convective heat transfer coefficient for surface n ($W \cdot m^{-2} \cdot K^{-1}$)
H_r Thermal transmittance due to radiation ($W \cdot K^{-1}$)
h_r Radiative heat transfer coefficient ($W \cdot m^{-2} \cdot K^{-1}$)
$\mathcal{J}_n$ Radiosity of surface n ($W \cdot m^{-2}$)
L_n Longwave radiant heat flux incident on surface n ($W \cdot m^{-2}$)
m Integer denoting particular surface
$\dot{m}_a$ Mass flow rate of air ($kg \cdot s^{-1}$)
N Total number of surfaces
N_v Number of room air changes (h–1)
n Integer denoting particular surface
R Radiant fraction of source from source
R_{sin} Thermal resistance between inner face of surface n and environmental temperature ($m^2 \cdot K \cdot W^{-1}$)
R_{sn} Thermal resistance of surface n ($m^2 \cdot K \cdot W^{-1}$)
U_n Thermal transmittance for material of which surface n is composed ($W \cdot m^{-2} \cdot K^{-1}$)
U_n' Thermal transmittance between inner face of surface n and heat transfer temperature on outer face of surface n ($W \cdot m^{-2} \cdot K^{-1}$)
U_p Thermal transmittance modified for heat flow through internal partition ($W \cdot m^{-2} \cdot K^{-1}$)
V Room volume (m^3)
α Surface absorption coefficient
ε_n Emissivity of surface n
Φ_{con} Convective energy from emitter (W)
Φ_f Fabric heat gain (W)
Φ_{ln} Longwave energy incident on surface n from sources other than room surfaces (W)
Φ_{rad} Radiant energy from emitter (W)
Φ_t Total heat loss (W)
ϕ_n Radiant heat flow from surface n ($W \cdot m^{-2}$)
θ_{ai} Inside air temperature (°C)
θ_{ain} Air temperature for convective heat exchange with surface n (°C)
θ_c Operative temperature at centre of room (°C)
θ_c' Operative temperature on far side of internal partition through which heat flow occurs (°C)
θ_{ei} Environmental temperature (°C)
θ_o External heat transfer temperature (°C)
θ_{on} External heat transfer temperature for surface n (°C)
θ_r Mean radiant temperature (°C)
θ_s Surface temperature (°C)
θ_{sn} Surface temperature of surface n (°C)
$\theta^{\star}$ Radiant-star temperature (°C)

5.A8.2 Full model

The rate of loss of heat from a space through the building fabric can be expressed as:

$$\Phi_f = \sum_{n=1}^{N} A_n (\theta_{sn} - \theta_{on}) / R_{sn} \tag{A8.1}$$

The rate of heat flow through a wall is equal to that into the wall, thus the fabric heat loss can also be expressed as:

$$\Phi_f = \sum_{n=1}^{N} [A_n (\theta_{ain} - \theta_{sn}) h_{cn} - \phi_n] \tag{A8.2}$$

Note that ϕ_n is positive for heat flows leaving the surface.

The first term inside the square brackets represents the rate of convection of heat from the room air to the surface and the second term is the rate of radiant heat flow into the surface. This radiant term represents the exchange of longwave radiation between the surface and all other surfaces within the room. (The calculation of steady state loss ignores shortwave radiation.) The exchange of longwave radiation can be seen as analogous to the reflection of light from a diffuse source, i.e. there are an infinite number of reflections of radiation between the surfaces.

The rate of radiant heat flow into the surface is the difference between that incident (L_n) upon the surface and that leaving the surface ($\mathcal{J}_n$) that is:

$$\phi_n = A_n (\mathcal{J}_n - L_n) \tag{A8.3}$$

Now the rate at which radiant energy leaves a surface may be expressed as:

$$\mathcal{J}_n = (1 - \varepsilon_n) L_n + \varepsilon_n E_{bn} \tag{A8.4}$$

Thus:

$$\phi_n = A_n (E_{bn} - \mathcal{J}_n) \varepsilon_n / (1 - \varepsilon_n) \tag{A8.5}$$

The radiation incident upon the surface is the sum of that received from other surfaces and that from radiant heating sources. Radiation from another surface depends upon the view factor between that surface and the subject surface (n) and the rate at which radiation leaves that surface (i.e. the radiosity).

Thus the radiation incident upon surface n is:

$$A_n L_n = \sum_{m=1}^{N} (\mathcal{J}_m A_m F_{m,n}) - \Phi_{ln} \tag{A8.6}$$

However:

$$A_n F_{n,m} = A_m F_{m,n} \tag{A8.7}$$

Therefore:

$$A_n L_n = \sum_{m=1}^{N} (\mathcal{J}_m A_n F_{n,m}) - \Phi_{\text{l}n} \qquad \text{(A8.8)}$$

This represents a set of simultaneous equations (one for each surface) that when solved give the amount of radiation leaving each surface ($\mathcal{J}_n$). Simultaneous solution means that the infinite number of reflections of radiation is accounted for automatically.

In order to solve equation A8.8 it is first necessary to substitute for L_n by combining equations A8.5 and A8.8. Assuming that $F_{n,m}$ is zero (i.e. that is all surfaces are planar), this results in the following equation set:

$$\mathcal{J}_n / (1 - \varepsilon_n) - \sum_{1}^{N} F_{n,m} \mathcal{J}_\text{m} \ldots$$

$$= \varepsilon_n E_{\text{b}n} / (1 - \varepsilon_n) + \Phi_{\text{l}n} / A_n \qquad \text{(A8.9)}$$

This relationship is converted into a heat loss model by linearising the black body emissive power and introducing the steady state surface heat balance. Thus:

$$E_{\text{b}n} = a + b\,\theta_{\text{s}n} \qquad \text{(A8.10)}$$

where a and b are constants.

From equation A8.1, for a single surface (n):

$$\Phi_\text{f} = A_n (\theta_{\text{s}n} - \theta_{\text{o}n})\, U_n' \qquad \text{(A8.11)}$$

Therefore, equating A8.11 and A8.2 to eliminate Φ_f gives:

$$-\phi_n + \theta_{\text{ai}n} h_{\text{c}n} + U_n' \theta_{\text{o}n} = \theta_{\text{s}n} (h_{\text{c}n} + U_n') \qquad \text{(A8.12)}$$

U_n' is the transmittance between the surface temperature $\theta_{\text{s}n}$ and the outside temperature $\theta_{\text{o}n}$, i.e. the heat transfer temperature on the other side surface n, given by:

$$U_n' = U_n / (1 - U_n R_{\text{si}n}) \qquad \text{(A8.13)}$$

$R_{\text{si}n}$ is the standard value of the inner surface resistance used to calculate the standard U-value (see chapter 3 of this Guide) for surface n, U_n. Since U_n' is also dependent on the external surface heat transfer coefficient, i.e. the surface coefficient appropriate to the 'other' side of surface, it may be necessary to include a correction for exposure.

Substitution of equations A8.10 and A8.12 into equation A8.9 gives the set of equations A8.14 (see below), which represent both radiant interchange between surfaces and the conduction of heat through room surfaces.

Equation A8.14 places no restrictions on the air temperature distribution within the space. A means of obtaining air temperatures would be to combine the solution of the above with computational fluid dynamics. Alternatively, some rules could be assigned to the distribution of air temperature throughout the space (Gagneau et al., 1997).

5.A8.3 Reference model

The reference model is developed by adding convective heat transfer and control sensor models to the full model and making some assumptions about the distribution of the radiant component of heat from the emitter.

The full model contains an arbitrary model of the convective heat transfer process. The reference model assumes a fully mixed space, i.e. the dry bulb temperature of the air does not vary from point to point within the space. Thus in equation A8.14 all values of $\theta_{\text{ai}n}$ are equal to the inside air temperature θ_ai and the convective heat balance is then given by:

$$-\sum_{n=1}^{N} h_{\text{c}n} A_n \theta_{\text{s}n} + \theta_\text{ai} (\dot{m}_\text{a} c_\text{p} + \sum_{n=1}^{N} h_{\text{c}n} A_\text{n})$$

$$= \Phi_\text{t} (1 - R) + \theta_\text{ao} \dot{m}_\text{a} c_\text{p} \qquad \text{(A8.16)}$$

The model is completed by the introduction of the control temperature (θ_c), for example the operative temperature which at low air speeds is the average of the air and mean radiant temperatures. The mean radiant temperature 'seen' by a sensor may be considered to be the equivalent temperature for radiant heat exchange between the sensor and its surroundings. It therefore depends upon:

- surface temperature
- surface emissivity
- emissivity of the sensor
- view factor between the surfaces and the sensor
- radiation from a heat emitter incident on the sensor.

Thus, the mean radiant temperature varies throughout the space. It is possible to model the sensor as an additional room surface. However, for the purposes of design calculations, the sensor is deemed to be located at a position where the proportion of longwave radiation received from each surface is directly proportional to the ratio of the area of the surface to the total room area. Furthermore, the sensor is also assumed to have an emissivity of unity (i.e. a black body). Thus the design mean radiant temperature is:

$$\theta_\text{r} = \frac{\Sigma\, \theta_{\text{s}n} A_n}{\Sigma A_n} + \frac{R\, \Phi_\text{t}}{h_\text{r} \Sigma A_n} \qquad \text{(A8.17)}$$

Note that h_r is calculated for an emissivity of unity.

$$(h_{\text{c}n} + U_n' + \varepsilon_n h_{\text{r}n})(\theta_{\text{s}n} / \varepsilon_\text{n}) - \sum_{m=1}^{N} (F_{n,m} / \varepsilon_\text{m}) [(h_{\text{c}m} + U_m')(1 - \varepsilon_m) + h_\text{r}\, \varepsilon_m]\, \theta_{\text{s}m} - (h_{\text{c}n}\, \theta_{\text{ai}n} / \varepsilon_n) + \sum_{m=1}^{N} (F_{n,m} / \varepsilon_\text{m}) [h_{\text{c}m} (1 - \varepsilon_m)\, \theta_{\text{ai}m}]$$

$$= (\theta_{\text{o}m} U_n' / \varepsilon_\text{n}) - \sum_{m=1}^{N} (F_{n,m} / \varepsilon_\text{m}) [U_m' (1 - \varepsilon_m)\, \theta_{\text{o}m}] + \Phi_{\text{l}n} / A_n \qquad \text{(A8.14)}$$

where:

$$h_{\text{r}n} = \varepsilon_\text{n}\, b_\text{n} \qquad \text{(A8.15)}$$

The control temperature is given by:

$$\theta_c = F_a\,\theta_{ai} + (1 - F_a)\,\theta_r \quad \text{(A8.18)}$$

where F_a = 0.5 if the sensed parameter is the operative temperature.

Assuming that any radiant heat input is uniformly distributed over each surface, and is equal to $(\Phi_t\,R / \Sigma A)$, the reference model may be represented by the equation set:

$$\boldsymbol{A}\,\boldsymbol{X} = \boldsymbol{C} \quad \text{(A8.19)}$$

where $\boldsymbol{A}$, $\boldsymbol{X}$ and $\boldsymbol{C}$ are matrices, as defined in the following boxes.

Matrix $\boldsymbol{A}$:

(*a*) Surface heat balance equations

Terms $A(n,n)$ for n = 1 to n = total number of room surfaces:

$$A(n,n) = (h_{cn} + U_n' + h_r\,\varepsilon_n) / \varepsilon_n \quad \text{(A8.20)}$$

Terms $A(n,m)$ where $n \neq m$, for n = 1 to n = total number of room surfaces and for m = 1 to m = total number of room surfaces:

$$A(n,m) = -F_{n,m}\,[(h_{cm} + U_m')\,(1 - \varepsilon_m) + h_r\,\varepsilon_m)] / \varepsilon_m \quad \text{(A8.21)}$$

Terms $A(n,m)$ for n = 1 to n = total number of room surfaces and for m = (total number of room surfaces + 1):

$$A(n,m) = (-h_{cn} / \varepsilon_n) + \sum_{i=1}^{N} h_{ci}\,F_{n,i}\,(1 - \varepsilon_n) / \varepsilon_i \quad \text{(A8.22)}$$

Terms $A(n,m)$ for n = 1 to n = total number of room surfaces and for m = (total number of room surfaces + 2):

$$A(n,m) = -R / \Sigma A \quad \text{(A8.23)}$$

(*b*) Control sensor heat balance equations

Terms $A(n,n)$ for n = total number of room surfaces + 1:

$$A(n,n) = F_a \quad \text{(A8.24)}$$

where F_a is the fraction of the air temperature detected by the sensor. (F_a = 0.5 for a sensor detecting operative temperature.)

Terms $A(n,m)$ for n = (total number of room surfaces + 1) and for m = 1 to m = total number of room surfaces:

$$A(n,m) = (1 - F_a)\,A_n / \Sigma A_n \quad \text{(A8.25)}$$

Terms $A(n,m)$ for n = (total number of room surfaces + 1) and for m = (total number of room surfaces + 2):

$$A(n,m) = R\,(1 - F_a) / (h_r\,\Sigma A) \quad \text{(A8.26)}$$

(*c*) Convection heat balance

Terms $A(n,n)$ for n = (total number of room surfaces + 2):

$$A(n,n) = (R - 1) / \Sigma A \quad \text{(A8.27)}$$

Terms $A(n,m)$ for n = (total number of room surfaces + 2) and for m = 1 to m = total number of room surfaces:

$$A(n,m) = -h_{cn}\,A_n / \Sigma A \quad \text{(A8.28)}$$

Terms $A(n,m)$ for n = (total number of room surfaces + 2) and for m = (total number of room surfaces + 1):

$$A(n,m) = [C_v + \Sigma\,(A_n\,h_{cn})] / \Sigma A \quad \text{(A8.29)}$$

Vector $\boldsymbol{C}$:

Terms $C(n)$ for n = 1 to n = the total number of room surfaces:

$$C(n) = (\theta_{on}\,U_n' / \varepsilon_n) - \sum_{i=1}^{N} [F_{n,i}\,U_n'\,\theta_{oi}\,(1 - \varepsilon_i) / \varepsilon_i \quad \text{(A8.30)}$$

Terms $C(n)$ for n = (total number of room surfaces + 1):

$$C(n) = \theta_c \quad \text{(A8.31)}$$

Terms $C(n)$ for n = (total number of room surfaces + 2):

$$C(n) = \theta_{ao}\,C_v / \Sigma A \quad \text{(A8.32)}$$

Solution vector $\boldsymbol{X}$:

Terms $X(n)$ for n = 1 to n = total number of room surfaces provide the temperatures for each surface.

Term $X(n)$ for n = (total number of room surfaces + 1) provides the room air temperature.

Term $X(n)$ for n = (total number of room surfaces + 2) provides the emitter output (i.e. sum of convective and radiant outputs).

The ventilation transmittance is represented by the conventional term C_v, see equation 5.34. If it is necessary to take account of air flows from a number of sources, that term in matrix $\boldsymbol{A}$ should be replaced by the summation $\Sigma\,(\dot{m}_a\,c_p)_i$ where the summation covers all sources i.

In vector $\boldsymbol{C}$, the term $(\theta_{ao}\,C_v / \Sigma A)$ is then replaced by $[\Sigma\,(\theta_i\,\dot{m}_a\,c_p)_i / \Sigma A]$ where θ_i is the temperature of air from source i.

View factors are not easy to calculate and while some standard relationships are given in chapter 3 of CIBSE Guide C (2007), these will not cover many applications. Figure 5.A8.1 and the following algorithm enables view factors to be determined for rectangular rooms (ASHRAE, 1976).

(*a*) Two parallel room surfaces

Radiation shape factor ($F_{1\text{-}2}$) between parallel surfaces 1 and 2 separated by a distance G, see Figure 5.A8.1(a), is given by:

$$2\,\pi\,(b_1 - a_1)\,(d_1 - c_1)\,F_{1\text{-}2} =$$
$$\{[P(b_2 - b_1) + P(a_2 - a_1)] \times [Q(c_2 - c_1) + Q\,(d_2 - d_1)$$
$$- Q(c_2 - d_1) - Q(d_2 - c_1)]\}$$
$$+ \{[P(b_2 - a_1) + P(a_2 - b_1)] \times [Q(c_2 - d_1)$$
$$+ Q(d_2 - c_1) - Q(c_2 - c_1) - Q(d_2 - d_1)]\} \qquad \text{(A8.33)}$$

P and Q are functions; expanding equation 5.97 gives products of the form $P(b_2 - b_1)\,Q(c_2 - c_1)$, given by:

$$P(Z_1)\,Q(Z_2) = Z_1\,W \tan^{-1}(Z_1 / W)$$
$$+ Z_2\,V \tan^{-1}(Z_2 / V) - (G^2 / 2) \ln\,[(W^2 + Z_1^{\,2}) / W_2)]$$
$$\text{(A8.34)}$$

where Z_1 and Z_2 are generalised variables, e.g. $Z_1 = (b_2 - b_1)$ and $Z_2 = (c_2 - c_1)$, and:

$$V^2 = G^2 + Z_{12} \qquad \text{(A8.35)}$$

$$W^2 = G^2 + Z_2^{\,2} \qquad \text{(A8.36)}$$

(*b*) Two perpendicular room surfaces

Radiation shape factor ($F_{1\text{-}2}$) between perpendicular surfaces 1 and 2, see Figure 5.A8.1(a), is given by:

$$2\,\pi\,(b_1 - a_1)\,(d_1 - c_1)\,F_{1\text{-}2} = \{[R(b_2 - b_1) + R(a_2 - a_1)]$$
$$\times [S(c_2 - c_1) + S(d_2 - d_1) - S(c_2 - d_1) - S(d_2 - c_1)]\}$$
$$+ \{[R(b_2 - a_1) + R(a_2 - b_1)] \times [S(c_2 - d_1)$$
$$+ S(d_2 - c_1) - S(c_2 - c_1) - S(d_2 - d_1)]\} \qquad \text{(A8.37)}$$

R and S are functions; expanding equation A8.37 gives products of the form $R(b_2 - b_1)\,S(c_2 - c_1)$, given by:

$$R(Z_1)\,S(Y_2 - Y_1) = T\,Z_1 \tan^{-1}(Z_1 / T)$$
$$+ {}^1/_4\,(Z_1^{\,2} - T^2) \ln\,(T^2 + Z_1^{\,2}) \qquad \text{(A8.38)}$$

where Z_1 and $(Y_2 - Y_1)$ are generalised variables, as above, and:

$$T^2 = Y_2^{\,2} + Y_1^{\,2} \qquad \text{(A8.39)}$$

These equations when combined with view factor algebra will satisfy the majority of needs. The relevant view factor algebra is as follows.

For conservation of energy:

$$\sum_{m=1}^{M} F_{n,m} = 1.0 \qquad \text{(A8.40)}$$

where the summation is over all surfaces comprising the enclosure.

For reciprocity:

$$A_n\,F_{n,m} = A_m\,F_{m,n} \qquad \text{(A8.41)}$$

If surface m is constructed from a number of sub-surfaces, e.g. windows, doors, wall, then:

$$A_n\,F_{n,m} = A_n\,F_{n,m1} + A_{\text{n}}\,F_{n,m2} + \ldots\ldots \qquad \text{(A8.42)}$$

where surface m is made up from sub-surfaces $m1$, $m2$ etc.

For cases where non-rectangular or concealed surfaces are involved or where rooms are not orthogonal, numerical techniques will be necessary for calculating view factors. These methods usually make use of contour integration (Walton, 1986) although statistically based methods have also been used (Malalasekera, 1993). The application of these methods is outside of the scope of this Guide.

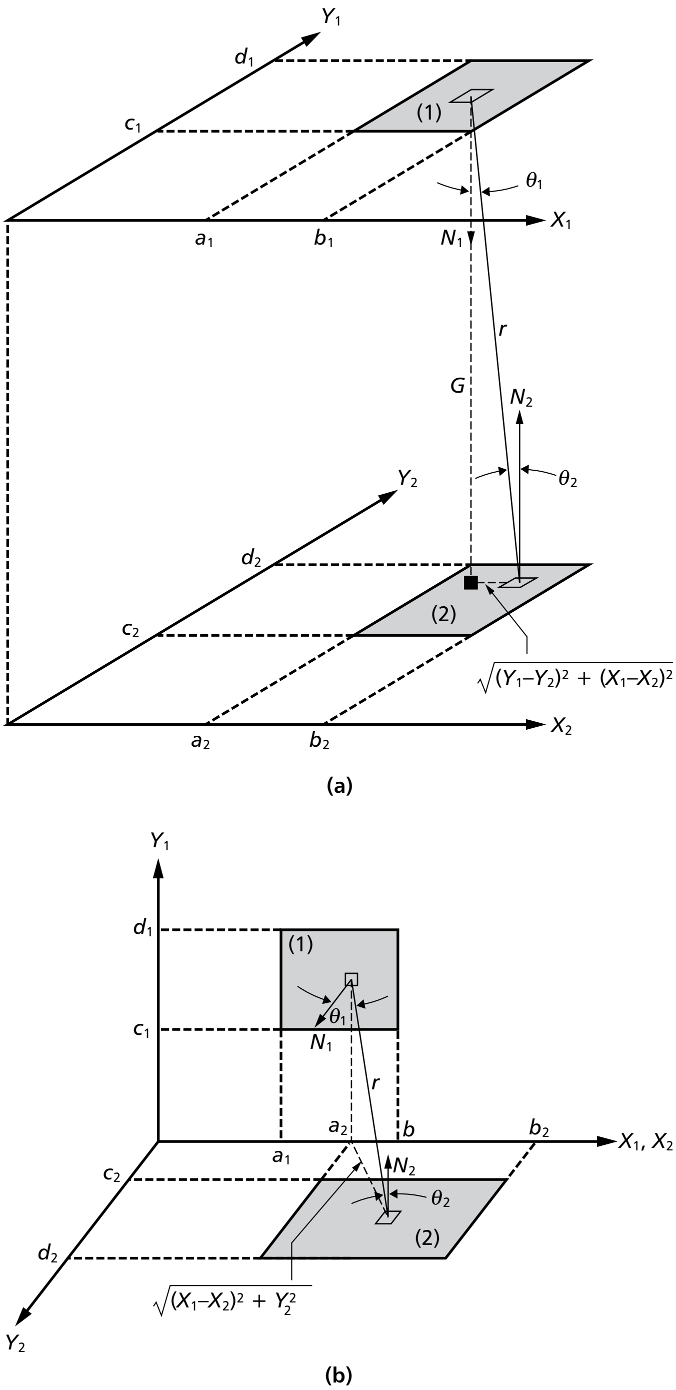

Figure 5.A8.1 View factors for radiation heat exchange; (a) between two parallel room surfaces, (b) between two perpendicular room surfaces

5.A8.4 Basic model

The reference model above considered the heat transfer process within a room from the view of direct surface-to-surface radiant heat flows, surface-to-air convection and surface-to-outside conduction. The surface-to-surface radiant flow is the most difficult of these processes to model. An alternative approach is to assume that just as all convective heat input must first increase the air temperature, i.e. enters the 'air temperature node' so all radiant heat enters at the 'radiant temperature node' ($\theta^\star$). Heat then flows into each room surface by means of a heat transfer coefficient that is adjusted to take account of the multiple reflections of radiation between surfaces. Davies (1990) has shown that it is possible to make a very close approximation to the exact equivalent of the radiosity matrix used in the reference model for a six-sided enclosure.

In such a case the radiant heat transfer coefficient is equal to the product ($E^\star_n\, h_r$) where:

$$E^\star_n = \varepsilon_n / (1 - \varepsilon_n + \beta_n\, \varepsilon_n) \quad \text{(A8.43)}$$

where β_n is given by the regression equation:

$$\beta_n = 1 - f_n\, [1 + 3.53\, (f_n - 0.5) - 5.04\, (f_n^2 - 0.25)] \quad \text{(A8.44)}$$

where:

$$f_n = A_n / \Sigma A \quad \text{(A8.45)}$$

The standard error for the regression is 0.0068 and the exact value of β_n for a cube is 5/6.

While the terms β_n are specific to a six-sided enclosure, they can often be used in most design applications. Introducing this close approximation to the radiant exchange process, greatly simplifies matrix $\boldsymbol{A}$ at the minor expense of the introduction of a new temperature $\theta^\star$, as follows:

$$(\sum_{n=1}^{N} h_{cn} A_n + \dot{m}_a c_p)\, \theta_{ai} - \sum_{n=1}^{N} h_{cn} A_n\, \theta_{sn} = \Phi_{con} + \theta_{ao}\, \dot{m}_a c_p \quad \text{(A8.46)}$$

$$\sum_{n=1}^{N} h^\star_r E^\star_n A_n\, \theta^\star - \sum_{n=1}^{N} h^\star_r E^\star_n A_n\, \theta_{sn} = \Phi_n + \Phi_{rad} \quad \text{(A8.47)}$$

where Φ_{con} is the convective output from an emitter (W) and Φ_{rad} is the radiant output (W).

For each surface n:

$$-A_n h_{cn}\, \theta_{ai} - A_n E^\star_n h_r\, \theta^\star + (h_{cn} + E^\star_n h_r + U_n')\, A_n\, \theta_{sn}$$
$$= \Phi_n + \theta_{on} A_n U_n' \quad \text{(A8.48)}$$

where Φ_n is a heat input to surface n (W), e.g. the absorbed solar radiation incident upon the surface. For the purposes of a heat loss model, all Φ_n are set to zero.

The basic model is given by the equation set represented by the matrix equation:

$$\boldsymbol{A}^\star \boldsymbol{X}^\star = \boldsymbol{C}^\star \quad \text{(A8.49)}$$

where $\boldsymbol{A}^\star$, $\boldsymbol{X}^\star$ and $\boldsymbol{C}^\star$ are matrices, as defined in the following boxes.

Matrix $\boldsymbol{A}^\star$:

(*a*) Surface heat balance

Terms $A^\star(n,n)$ for $n = 1$ to n = total number of room surfaces:

$$A^\star(n,n) = (h_{cn} + U_n' + E^\star_n h_r) \quad \text{(A8.50)}$$

Terms $A^\star(n,m)$ where $n \neq m$, for $n = 1$ to n = total number of room surfaces and for $m = 1$ to m = total number of room surfaces:

$$A^\star(n,m) = 0 \quad \text{(A8.51)}$$

Terms $A^\star(n,m)$ for $n = 1$ to n = total number of room surfaces and for m = (total number of room surfaces + 1):

$$A^\star(n,m) = -h_{cn} \quad \text{(A8.52)}$$

Terms $A^\star(n,m)$ for $n = 1$ to n = total number of room surfaces and for m = (total number of room surfaces + 2):

$$A^\star(n,m) = 0 \quad \text{(A8.53)}$$

Terms $A^\star(n,m)$ for $n = 1$ to n = total number of room surfaces and for m = (total number of room surfaces + 3):

$$A^\star(n,m) = -E^\star_n h_r \quad \text{(A8.54)}$$

(*b*) Control sensor heat balance

Terms $A^\star(n,n)$ for n = (total number of room surfaces + 1):

$$A^\star(n,n) = F_a \quad \text{(A8.55)}$$

Terms $A^\star(n,m)$ for n = (total number of room surfaces + 1) and for $m = 1$ to m = total number of room surfaces:

$$A^\star(n,m) = \varepsilon_m\, (1 - F_a)\, A_m / \Sigma\, (A\, \varepsilon) \quad \text{(A8.56)}$$

Terms $A^\star(n,m)$ for n = (total number of room surfaces + 1) and for m = (total number of room surfaces + 2):

$$A^\star(n,m) = R\, (1 - F_a) / (h_r\, \Sigma A) \quad \text{(A8.57)}$$

Terms $A^\star(n,m)$ for n = (total number of room surfaces + 1) and for m = (total number of room surfaces + 3):

$$A^\star(n,m) = 0 \quad \text{(A8.58)}$$

(*c*) Convection heat balance

Terms $A^\star(n,m)$ for n = (total number of room surfaces + 2) and for $m = 1$ to m = total number of room surfaces:

$$A^\star(n,m) = -h_{cn} A_m / \Sigma\, (A\, \varepsilon) \quad \text{(A8.59)}$$

Terms $A^\star(n,m)$ for n = (total number of room surfaces + 2) and for m = (total number of room surfaces + 1):

$$A^\star(n,m) = [C_\text{v} + \Sigma\,(A_m\,h_{cm})] \,/\, \Sigma\,(A\,\varepsilon) \qquad \text{(A8.60)}$$

Terms $A^\star(n,m)$ for n = (total number of room surfaces + 2) and for m = (total number of room surfaces + 2):

$$A^\star(n,m) = (R - 1) \,/\, \Sigma A \qquad \text{(A8.61)}$$

Terms $A^\star(n,m)$ for n = (total number of room surfaces + 2) and for m = (total number of room surfaces + 3):

$$A^\star(n,m) = 0 \qquad \text{(A8.62)}$$

(*d*) Radiant heat balance

Terms $A^\star(n,n)$ for n = (total number of room surfaces + 3):

$$A^\star(n,n) = \Sigma\, E_n^\star\, h_\text{r}\, A_\text{n} \,/\, \Sigma A \qquad \text{(A8.63)}$$

Terms $A^\star(n,m)$ for n = (total number of room surfaces + 3) and for m = 1 to m = total number of room surfaces:

$$A^\star(n,m) = -E_m^\star\, h_\text{r}\, A_\text{m} \,/\, \Sigma A \qquad \text{(A8.64)}$$

Terms $A^\star(n,m)$ for n = (total number of room surfaces + 3) and for m = (total number of room surfaces + 1):

$$A^\star(n,m) = 0 \qquad \text{(A8.65)}$$

Terms $A^\star(n,m)$ for n = (total number of room surfaces + 3) and for m = (total number of room surfaces + 2):

$$A^\star(n,m) = -R \,/\, \Sigma A \qquad \text{(A8.66)}$$

Vector $\boldsymbol{C}^\star$:

Terms $C^\star(n)$ for n = 1 to n = the total number of room surfaces:

$$C^\star(n) = \theta_{on}\, U_\text{n}' \qquad \text{(A8.67)}$$

Terms $C^\star(n)$ for n = (total number of room surfaces + 1):

$$C^\star(n) = \theta_\text{c} \qquad \text{(A8.68)}$$

Terms $C^\star(n)$ for n = (total number of room surfaces + 2):

$$C^\star(n) = \theta_\text{ao}\, C_\text{v} \,/\, \Sigma A \qquad \text{(A8.69)}$$

Terms $C^\star(n)$ for n = total number of room surfaces + 3:

$$\text{C}^\star(n) = 0 \qquad \text{(A8.70)}$$

Solution vector $\boldsymbol{X}^\star$:

Terms $X^\star(n)$ for n = 1 to n = total number of room surfaces provide the temperature of each surface.

Term $X^\star(n)$ for n = (total number of room surfaces + 1) provides the room air temperature.

Term $X^\star(n)$ for n = (total number of room surfaces + 2) provides the total heat input.

Term $X^\star(n)$ for n = (total number of room surfaces + 3) provides the radiant-star temperature ($\theta^\star$), which is not the radiant temperature.

The ventilation transmittance is represented by the conventional term C_v, see equation 5.34. If it is necessary to take account of air flows from a number of sources, that term in matrix $\boldsymbol{A}$ should be replaced by the summation $\Sigma\,(\dot{m}_\text{a}\, c_\text{p})_i$ where the summation covers all sources i.

In vector $\boldsymbol{C}$, the term $(\theta_\text{ao}\, C_\text{v} \,/\, \Sigma\, A)$ is then replaced by $[\Sigma\,(\theta_i\, \dot{m}_\text{a}\, c_\text{p})_i \,/\, \Sigma\, A]$ where θ_i is the temperature of air from source i.

5.A8.5 Simple model

If the radiant exchange between surfaces can be treated separately, the surface heat balance equations are de-coupled and the need for matrix manipulation is removed. This leads to a manual calculation procedure.

One means of achieving this approximation is to assume that, with the exception of the subject surface, all surface temperatures are known. In this case, the heat balance on the subject surface is described by the surface heat balance equations given for the basic model, see equation A8.48. Hence:

$$\theta_\text{s}\,(h_\text{c} + U' + E^\star\, h_\text{r}) - h_\text{c}\,\theta_\text{ai} - E^\star\, h_\text{r}\,\theta^\star = \theta_\text{o}\, U' \qquad \text{(A8.71)}$$

Rearranging equation A8.71 gives the fabric heat loss:

$$U'\,(\theta_\text{s} - \theta_\text{o}) = h_\text{c}\,(\theta_\text{ai} - \theta_\text{s}) + h_\text{r}\, E^\star\,(\theta^\star - \theta_\text{s}) \qquad \text{(A8.72)}$$

It then remains to determine a value for $E^\star$. A simple method should use parameters that are independent of the shape of the enclosure. The simplest assumption is that the subject surface has an area equivalent to one sixth of that of the enclosure of which it forms a part. Therefore, from equation A8.43, with $f_\text{n} = 1/6$ (see equation A8.49) and $\beta_\text{n} = 5/6$ (see equation A8.44):

$$E^\star = \frac{\varepsilon}{(1 - \varepsilon + 5/6\,\varepsilon)} \qquad \text{(A8.73)}$$

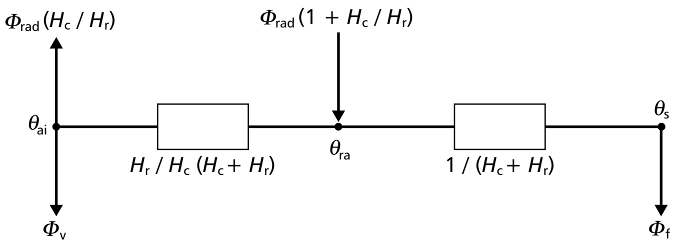

Figure 5.A2.2 Simplified heat flow network

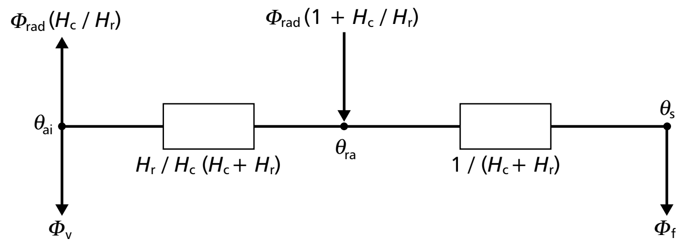

Figure 5.A2.3 Equivalent heat flow network

For $\varepsilon = 1, E^\star = {}^6/_5\,\varepsilon$:

$$\Phi_{\rm f} = h_{\rm c}\,(\theta_{\rm ai} - \theta_{\rm s}) + {}^6/_5\,\varepsilon\,h_{\rm r}\,(\theta^\star - \theta_{\rm s}) \tag{A8.74}$$

Equation A8.74 may be summed for all surfaces to give the total fabric loss, that is:

$$\Phi_{\rm f} = h_{\rm c}\,\Sigma A\,(\theta_{\rm ai} - \theta_{\rm m}) + {}^6/_5\,\varepsilon\,h_{\rm r}\,\Sigma A\,(\theta^\star - \theta_{\rm m}) \tag{A8.75}$$

where it is assumed that $h_{\rm c}$ and $h_{\rm r}$ are constants and that:

$$\theta_{\rm m} = \Sigma A\,\theta_{\rm s} / \Sigma A \tag{A8.76}$$

The heat input to a space comprises a radiant and convective component. From equation A8.47, the radiant component is:

$$\Phi_{\rm rad} = {}^6/_5\,\varepsilon\,h_{\rm r}\,\Sigma A\,(\theta^\star - \theta_{\rm m}) \tag{A8.77}$$

The convective components associated with the fabric heat loss is:

$$\Phi_{\rm con} = h_{\rm c}\,\Sigma A\,(\theta_{\rm ai} - \theta_{\rm m}) \tag{A8.78}$$

Equations A8.79, A8.77 and A8.78 can be expressed in analogue form by the network shown in Figure 5.A8.2, where:

$$H_{\rm c} = h_{\rm c}\,\Sigma A \tag{A8.79}$$

and:

$$H_{\rm r} = {}^6/_5\,\varepsilon\,h_{\rm r}\,\Sigma A \tag{A8.80}$$

Figure 5.A8.2 shows a radiant input $\Phi_{\rm rad}$ acting at the radiant star node $\theta^\star$, being lost by conduction $\Phi_{\rm f}$ from $\theta_{\rm s}$ and by ventilation $\Phi_{\rm v}$ from $\theta_{\rm ai}$. This network may be transformed exactly into that shown in Figure 5.A8.3 where the rad-air node $\theta_{\rm ra}$ is located on the convective transmittance $H_{\rm c}$, dividing it into two components: $X = H_{\rm c}\,(H_{\rm c} + H_{\rm r}) / H_{\rm r}$ and $Y = (H_{\rm c} + H_{\rm r})$. An augmented flow, $\Phi_{\rm rad}\,(1 + H_{\rm c} / H_{\rm r})$ acts at $\theta_{\rm ra}$ and the excess, $\Phi_{\rm rad}\,(H_{\rm c} / H_{\rm r})$ is withdrawn from $\theta_{\rm ai}$. Components X and Y can be considered, in effect, in parallel (Davies, 1990). The physically significant quantities, i.e. the observable temperatures $\theta_{\rm s}$ and $\theta_{\rm ai}$, and the heat flows from them, $\Phi_{\rm f}$ and $\Phi_{\rm v}$, can be considered the same in both cases.

There is a further transmittance, $[(H_{\rm c} + H_{\rm r})\,H_{\rm c} / H_{\rm r}]$, between $\theta_{\rm ra}$ and $\theta_{\rm ai}$. The rad-air temperature, $\theta_{\rm ra}$, is related to the two generating temperatures by the following equation:

$$\theta_{\rm ra} = \frac{H_{\rm c}\,\theta_{\rm ai}}{H_{\rm c} + H_{\rm r}} + \frac{H_{\rm r}\,\theta^\star}{H_{\rm c} + H_{\rm r}} \tag{A8.81}$$

If the mean surface temperature, $\theta_{\rm m}$, is taken as an approximation for $\theta^\star$, then $\theta_{\rm ra} \approx \theta_{\rm ei}$. Hence:

$$\theta_{\rm ei} = \frac{H_{\rm c}\,\theta_{\rm ai}}{H_{\rm c} + H_{\rm r}} + \frac{H_{\rm r}\,\theta_{\rm m}}{H_{\rm c} + H_{\rm r}} \tag{A8.82}$$

It is appropriate to standardise the heat transfer coefficients as follows:

$h_{\rm c} = 3.0$ W·m^{-2}·K^{-1} (an average figure)

$h_{\rm r} = 5.7$ W·m^{-2}·K^{-1} (for temperatures ≈ 20 °C)

$H_{\rm r} / \Sigma A = 6.0$ W·m^{-2}·K^{-1} (for $\varepsilon = 0.9$)

$H_{\rm c} / \Sigma A = 3.0$ W·m^{-2}·K^{-1}

Also:

$H_{\rm a} / \Sigma A = (H_{\rm r} + H_{\rm c})\,H_{\rm c} / H_{\rm r} = 4.5$ W·m^{-2}·K^{-1}

So:

$h_{\rm a} = 4.5$ W·m^{-2}·K^{-1}

Therefore, it follows from equation A8.82 that:

$$\theta_{\rm ei} = {}^1/_3\,\theta_{\rm ai} + {}^2/_3\,\theta_{\rm m} \tag{A8.83}$$

That is, the effective radiant heat input is 1.5 times the actual input, with the excess (50%) of radiant input subtracted from the convective component of the heat input. A further implication is that a heat source that is effectively directly linked to the environmental temperature has the characteristics of ²/₃ radiation and ¹/₃ convection.

It is accepted that a number of approximations are embodied in this relationship. However, empirical testing over a number of years has not revealed any serious deficiencies in practice and, as such, it is therefore accepted as the basis of the CIBSE simple heat loss model, which is developed as follows.

The heat loss due to the fabric is defined as:

$$\Phi_{\rm f} = \sum_{n=1}^{N} A_{\rm n}\,U_{\rm n}\,(\theta_{\rm ei} - \theta_{on}) \tag{A8.84}$$

Where the fabric term contains heat loss through internal partitions, a modified U-value should be used:

$$U_{\rm p} = \frac{U\,(\theta_{\rm c} - \theta_{\rm c}')}{(\theta_{\rm c} - \theta_{\rm ao})} \tag{A8.85}$$

This correction is based on the internal design operative temperature ($\theta_{\rm c}$) and therefore is not exact. However, the operative is usually very close to the heat loss temperature ($\theta_{\rm ei}$) which means that any error is small in what is already a second order correction. This approximation makes it unnecessary to determine the value of the environmental temperature in adjacent spaces.

The heat loss due to infiltration and/or ventilation by outdoor air is:

$$\Phi_{\rm v} = \frac{c_{\rm p}\,\rho\,N_{\rm v}\,V}{3600}\,(\theta_{\rm ai} - \theta_{\rm ao}) \tag{A8.86}$$

For practical purposes $(c_{\rm p}\,\rho / 3600) = {}^1/_3$, therefore $C_{\rm v} = N_{\rm v}\,V / 3$.

Hence:

$$\Phi_{\mathrm{v}} = C_{\mathrm{v}}\,(\theta_{\mathrm{ai}} - \theta_{\mathrm{ao}}) \qquad \text{(A8.87)}$$

Ventilation rates must include infiltration, natural ventilation due to open windows and, where appropriate, mechanical ventilation. Guidance on ventilation requirements and design allowances for infiltration are given in chapter 1 and chapter 4 of this Guide, respectively.

The total heat loss is the sum of the fabric and infiltration losses:

$$\Phi_{\mathrm{t}} = \sum_{n=1}^{N} A_n\,U_n\,(\theta_{\mathrm{ei}} - \theta_{\mathrm{o}n}) + C_{\mathrm{v}}\,(\theta_{\mathrm{ai}} - \theta_{\mathrm{ao}}) \qquad \text{(A8.88)}$$

For winter heating design conditions it is conventional to assume that the outside heat transfer temperature ($\theta_{\mathrm{o}n}$) equals the outside air temperature (θ_{ao}), therefore:

$$\Phi_{\mathrm{t}} = \sum_{n=1}^{N} A_n\,U_n\,(\theta_{\mathrm{ei}} - \theta_{\mathrm{ao}}) + C_{\mathrm{v}}\,(\theta_{\mathrm{ai}} - \theta_{\mathrm{ao}}) \qquad \text{(A8.89)}$$

In order to relate the heat loss to the design operative temperature, it is necessary to eliminate θ_{ai} and θ_{ei}. This is achieved by introducing factors $F_{1\mathrm{cu}}$ and $F_{2\mathrm{cu}}$, as follows (see Appendix 5.A2, equations A2.17 and A2.18:

$$F_{1\mathrm{cu}} = \frac{3.0\,(C_{\mathrm{v}} + 6\,\Sigma A)}{\Sigma\,(A\,U) + 18\,\Sigma A + 1.5\,R\,[3\,C_{\mathrm{v}} - \Sigma\,(A\,U)]} \qquad \text{(A8.90)}$$

$$F_{2\mathrm{cu}} = \frac{\Sigma\,(A\,U) + 18\,\Sigma A}{\Sigma\,(A\,U) + 18\,\Sigma A + 1.5\,R\,[3\,C_{\mathrm{v}} - \Sigma\,(A\,U)]} \qquad \text{(A8.91)}$$

Therefore the simple model is:

$$\Phi_{\mathrm{t}} = (F_{1\mathrm{cu}} \sum_{n=1}^{N} A_n\,U_n + F_{2\mathrm{cu}}\,C_{\mathrm{v}})\,(\theta_{\mathrm{c}} - \theta_{\mathrm{ao}}) \qquad \text{(A8.92)}$$

and the corresponding air temperature is calculated using the following equation (see Appendix 5.A2, equation A2.20):

$$\bar{\theta}_{\mathrm{ai}} = \frac{\bar{\Phi}_{\mathrm{t}}\,(1 - 1.5\,R) + C_{\mathrm{v}}\,\bar{\theta}_{\mathrm{ao}} + 6.0\,\Sigma A\,\bar{\theta}_{\mathrm{c}}}{C_{\mathrm{v}} + 6.0\,\Sigma A} \qquad \text{(A8.93)}$$

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